Robust Passification and Control of Non-Passive Systems

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1998 American Control Conference
Philadelphia, PA
June 24-26, 1998

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Abstract

This paper addresses the use of passivity-based techniques to obtain *robust* controller design for non-passive uncertain systems. It extends the previous results on passivity-based control of non-passive systems to include robustness of passification in the presence of plant uncertainty. In particular, sufficient conditions for robust passification are obtained for different passification methods in the presence of various types of plant uncertainties. These conditions can be used either to check the robustness of a controller designed for a nominal plant model or to perform iterative controller design to meet certain robustness criteria. The plant uncertainty models used include additive, multiplicative, and feedback uncertainty. For each of these uncertainty models, conditions for robust passification are derived for three different passification methods, namely, series, feedback, and feedforward passification.

1 Passivity-based Control of Non-Passive Systems

A large class of physical systems can be classified as being naturally passive. Examples of such systems include flexible space structures with collocated and compatible actuators and sensors. Robust stabilization and control of such systems has received considerable attention in the literature, and a number of stability results exist in that area. The most basic of the stability results states that the negative feedback interconnection of two passive systems is Lyapunov-stable. Numerous other stability results exist which are essentially different variations of this basic result. The least restrictive result for linear, time-invariant (LTI) systems states that the feedback interconnection of a positive-real (PR) system and a marginally strict positive-real (MSPR) system is asymptotically stable [Jos.96]. Some nonlinear extensions of these results were obtained in [Jos.96a; Kel.96]. Passivity-based controllers have proved to be highly effective in robustly controlling inherently passive linear and nonlinear plants such as, flexible space structures or multilink flexible robots [Kel.96, Jos.89]. The passivity of such systems is model-independent; therefore, passivity-based controllers are robust to modeling errors and parametric uncertainties. Most physical systems, however, are not inherently passive, and passivity-based control techniques cannot be used directly for such systems. One method of making non-passive systems amenable to passivity-based control is to *passify* such systems (i.e., rendering system passive) using suitable compensation. If the compensated system is *robustly passive* despite plant uncertainties, it can be robustly stabilized by any MSPR controller. In [Kel.97] various passification techniques were presented and some numerical examples were given for demonstrating the use of such techniques. A brief overview of passification methods presented in [Kel.97] is given in the next subsection.

1.1 Passification Methods

Four different passification methods, series, feedback, feedforward, and hybrid, passification, were given in [Kel.97] for finite-dimensional linear, time-invariant non-passive systems. The idea of series passification is to design a series compensator $C_s(s)$ for the non-passive plant $P(s)$ such that the compensated plant $P_c(s) = C_s(s)P(s)$ (or $P_c(s) = P(s)C_s(s)$) is positive-real. Similarly, in feedback passification the objective is to design a feedback compensator $C_f(s)$ such that the closed-loop system $P(s)[I + P(s)C_f(s)]^{-1}$ is positive-real. Similar approaches can be taken in the other two methods of passification. Two important classes of non-passive systems which demand more attention are open-loop unstable systems and nonminimum-phase systems. In the case of open-loop unstable systems, the first step in passification is to stabilize the system using feedback compensation and then use, if necessary, additional compensation to render the stabilized plant positive-real. Similarly, for nonminimum-phase systems, the first step in passification is to render the system minimum-phase by feedforward compensation and then use additional compensation if necessary to render the resulting minimum-phase plant positive-real. Once passified, the system can be controlled by any marginally strict positive-real (MSPR) or weakly SPR (WSPR) controller [Jos.96, Loz.90]. Methods for designing WSPR controllers that are optimal in the linear-quadratic-Gaussian (LQG) sense or satisfy an H_∞ performance bound, were discussed in [Loz.90], [Had.94]. One important thing to be noted here is that in the case of inherently passive systems, the use of an MSPR controller gurantees stability robustness to unmodeled dynamics and parametric uncertainties; however, in the case of non-passive systems which are rendered passive using passifying compensation, the stability robustness depends on robustness

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of passification. That is, the problem of *robust stability* is transformed into the problem of *robust passification.* It is therefore necessary to develop the mathematical framework to address this problem. In this paper, a number of analytical sufficient conditions are derived to check the robustness of passification. In the next section we present a series of theorems which give conditions under which passifying compensators would be robust in the presence of different types of plant uncertainties.

2 Robustness of Passification

In deriving the robust passification conditions, we have considered three types of passifying compensators: series, feedback, and feedforward, and three types of plant uncertainty models: additive, multiplicative, and feedback. Thus, in total, we have nine different conditions given below. The proofs, which are based on the definition of positive-realness of the transfer function, are omitted due to space limitations. $P(s)$, $K(s)$, and $\Delta(s)$ denote $m \times m$ transfer function matrices of the nominal plant, the passifying compensator, and the uncertainty, respectively. (That is, the class of plants considered here has equal number of inputs and outputs). In the theorem statements, * denotes the conjugate transpose and the argument $j\omega$ has been omitted for brevity. All conditions presented in the following theorems require that the nominal passified plant is at least WSPR.

2.1 Series passification

It is assumed in this section that a pre-compensator $K(s)$ is used for series passification. However, similar results can be obtained for post-compensator, and for both pre- and post-compensators.

2.1.1 Additive uncertainty

Theorem 1 *Suppose a non-passive plant $P(s)$ is passified by a series compensator $K(s)$. Then a sufficient condition for robust passification in the presence of additive plant uncertainty, Δ, is given by*

$$\overline{\sigma}(\Delta) < \frac{\lambda_{min}(PK + K^*P^*)}{2\,\overline{\sigma}(K)} \quad \forall\omega \geq 0 \tag{1}$$

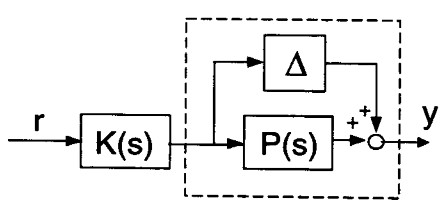

Figure 1: Series passification with additive uncertainty

2.1.2 Multiplicative uncertainty

Theorem 2 Suppose a non-passive plant $P(s)$ is passified by a series compensator $K(s)$. Then a sufficient condition for robust passification in the presence of multiplicative plant uncertainty, Δ, is given by

$$\overline{\sigma}(\Delta) < \frac{\lambda_{min}(PK + K^*P^*)}{2\,\overline{\sigma}(PK)} \quad \forall\omega \geq 0 \tag{2}$$

A similar condition can be obtained for multiplicative uncertainty at the output.

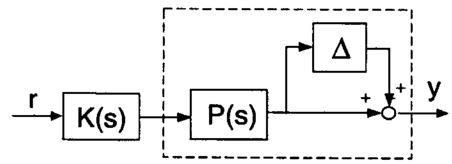

Figure 2: Series passification with multiplicative uncertainty

2.1.3 Feedback uncertainty

Theorem 3 Suppose a non-passive plant $P(s)$ is passified by a series compensator $K(s)$. Then a sufficient condition for robust passification in the presence of feedback plant uncertainty, Δ, is given by

$$\overline{\sigma}(\Delta) < \frac{\lambda_{min}(PK + K^*P^*)}{2\,\overline{\sigma}(PK)\overline{\sigma}(P)} \quad \forall\omega \geq 0 \tag{3}$$

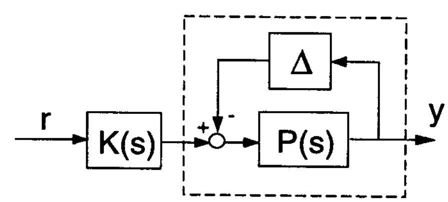

Figure 3: Series passification with feedback uncertainty

2.2 Feedback passification

2.2.1 Additive uncertainty

Theorem 4 Suppose a non-passive plant $P(s)$ is passified by a feedback compensator $K(s)$. Then a sufficient condition for robust passification in the presence of additive plant uncertainty, Δ, is given by

$$\overline{\sigma}(\Delta) < \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \forall\omega \geq 0 \tag{4}$$

where $a = \overline{\sigma}(K)$, $b = \overline{\sigma}(I + PK) + \overline{\sigma}(PK)$, $c = -\frac{\lambda_{min}(\overline{P}+\overline{P}^*)}{2}$, and $\overline{P} = P(I + PK)^*$, or (4) is satisfied with $a = \overline{\sigma}(\tilde{P})\overline{\sigma}(K + K^*)$, $b = 2\overline{\sigma}(\tilde{P})[1 + \overline{\sigma}(\overline{P}K^*)]$, $\tilde{P} = (I + PK)^{-1}$, $\overline{P} = \tilde{P}P$.

2.2.2 Multiplicative uncertainty

Theorem 5 Suppose a non-passive plant $P(s)$ is passified by a feedback compensator $K(s)$. Then a sufficient condition

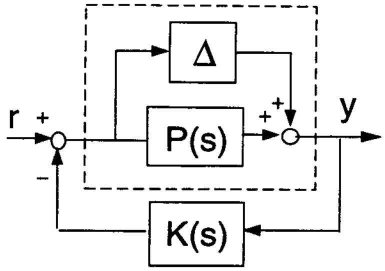

Figure 4: Feedback passification with additive uncertainty

for robust passification in the presence of multiplicative plant uncertainty, Δ, is given by

$$\overline{\sigma}(\Delta) < \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \forall\omega \geq 0 \tag{5}$$

where $a = \overline{\sigma}[P(K+K^*)P^*]$, $b = 2(\overline{\sigma}(P)+\overline{\sigma}[P(K+K^*)P^*]$, $c = -\frac{\lambda_{min}(\overline{P}+\overline{P}^*)}{2}$, *and* $\overline{P} = P(I+PK)^*$, *or if (5) is satisfied with* $a = 2\overline{\sigma}^2(\tilde{P})\overline{\sigma}^2(P)\overline{\sigma}(K)$, $b = 2\overline{\sigma}(\tilde{P})\overline{\sigma}[P(I+K\overline{P}]$, $\tilde{P} = (I+PK)^{-1}$, $\overline{P} = \tilde{P}P$.

A similar condition can be obtained for multiplicative uncertainty at the output.

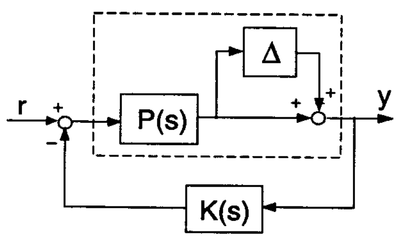

Figure 5: Feedback passification with multiplicative uncertainty

2.2.3 Feedback uncertainty

Theorem 6 *Suppose a non-passive plant $P(s)$ is passified by a feedback compensator $K(s)$. Then a sufficient condition for robust passification in the presence of feedback plant uncertainty, Δ, is given by*

$$\overline{\sigma}(\Delta) < \frac{\lambda_{min}(\overline{P}+\overline{P}^*)}{2\ (\overline{\sigma}(P))^2} \quad \forall\omega \geq 0 \tag{6}$$

where $\overline{P} = P(I+PK)^*$ *or if (6) is satisfied with* $\overline{P} = (I+PK)^{-1}P$.

2.3 Feedforward passification

2.3.1 Additive uncertainty

Theorem 7 *Suppose a non-passive plant $P(s)$ is passified by a feedforward compensator $K(s)$. Then a sufficient condition for robust passification in the presence of additive plant uncertainty, Δ, is given by*

$$\overline{\sigma}(\Delta) < \frac{\lambda_{min}(\overline{P}+\overline{P}^*)}{2} \quad \forall\omega \geq 0 \tag{7}$$

where $\overline{P} = P+K$.

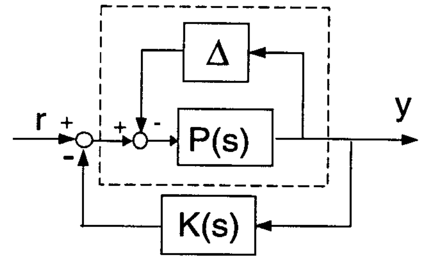

Figure 6: Feedback passification with feedback uncertainty

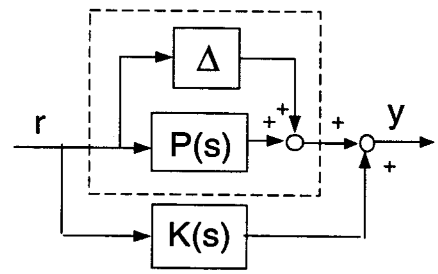

Figure 7: Feedforward passification with additive uncertainty

2.3.2 Multiplicative uncertainty

Theorem 8 *Suppose a non-passive plant $P(s)$ is passified by a feedforward compensator $K(s)$. Then a sufficient condition for robust passification in the presence of multiplicative plant uncertainty, Δ, is given by*

$$\overline{\sigma}(\Delta) < \frac{\lambda_{min}(\overline{P}+\overline{P}^*)}{2\ \overline{\sigma}(P)} \quad \forall\omega \geq 0 \tag{8}$$

where $\overline{P} = P+K$.

A similar condition can be obtained for multiplicative uncertainty at the output.

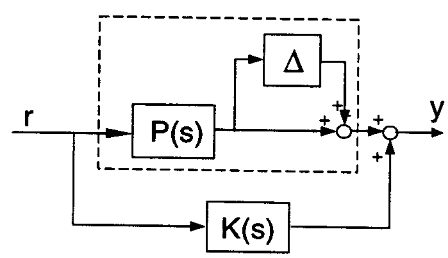

Figure 8: Feedforward passification with multiplicative uncertainty

2.3.3 Feedback uncertainty

Theorem 9 *Suppose a non-passive plant $P(s)$ is passified by a feedforward compensator $K(s)$. Then a sufficient condition for robust passification in the presence of feedback plant uncertainty, Δ, is given by*

$$\overline{\sigma}(\Delta) < \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \forall\omega \geq 0 \tag{9}$$

where $a = \overline{\sigma}^2(P)\overline{\sigma}(K + K^*)$, $b = 2[\overline{\sigma}(P + K^*)\overline{\sigma}(P)]$, $c = -\lambda_{min}(\overline{P} + \overline{P}^*)$, *and* $\overline{P} = P + K$.

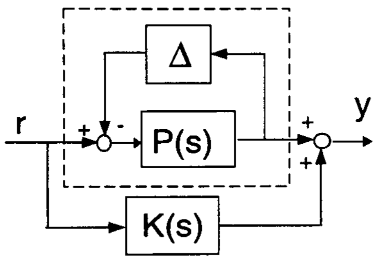

Figure 9: Feedforward passification with feedback uncertainty

3 Numerical Examples

In this section, we present two numerical examples to demonstrate the use of passivity-based robust control design techniques for uncertain non-passive systems. In the first example, we consider a long planar elastic beam with free-free boundary conditions. The problem is to control the rotation of the beam using a pair of non-collocated torque actuator and rate sensor. The truth model consists of one rigid body mode and the first five elastic modes. The design model includes only the rigid body plus first two elastic modes. The remaining three elastic modes were modeled as additive plant uncertainty. The first five natural frequencies of the free-free modes are (in rad/sec): 0.428, 1.172, 2.297, 3.797, and 5.672. The damping ratio for all five modes was assumed to be 0.028. The nominal plant model, which is not passive, was passified using a simple first order feedback compensator with pole at -100 and dc gain of 10. The robustness of passification to the unmodeled dynamics (consisting of last three elastic modes) was then checked by using the sufficient condition derived in Theorem 4. As seen in Figure 10, the robust passification condition was satisfied for additive uncertainty model consisting of the last three elastic modes. (The $\overline{\sigma}(M)$ in Fig. 10 represents the right hand side of Eq. (4)). The robustness of passification for simultaneous, parametric and unmodeled, uncertainty was also checked by assuming $\pm 10\%$ variation in the first two elastic mode frequencies. It was observed that the plant remains robustly passive under such simultaneous perturbations (Figure 11). However, the sufficient condition of Theorem 4 is violated, i.e., the $\overline{\sigma}(\Delta)$ curve does not remain below $\overline{\sigma}(M)$ curve for certain frequency range. However, it is to be noted that the additive uncertainty model is conservative when used for modeling parametric perturbations.

In the second example, we consider a longitudinal dynamic model of an F-18 HARV configuration [Ost.94]. The HARV configuration is a modified version of an F-18 airplane model which includes multi-axis thrust vectoring capability for pitch and yaw control power. The longitudinal models for pitch-axis control of HARV for four different flight conditions at the altitude of $15,000$ ft are considered as focus configurations for controller design. The four configurations had the following combinations of speed and normal acceleration: (1) 0.7 Mach and 1g, (2) 0.6 Mach and 1g, (3) 0.49 Mach and 1g, and (4) 0.3 Mach and 0.37g. The control input to the plant is elevator deflection and the output is the pitch rate. The controller design was obtained based on a nominal 4th-order plant model for the second flight condition, i.e., altitude of $15,000$ ft, speed 0.6 Mach, and acceleration of 1g. The passification of the nominal plant model was achieved by using a proper, third-order series compensator with poles at -10, -.05, and $-.0035$ and zeros at -1, -0.5, and -0.08. The passification was chosen so as to be robust to mach number variation between 0.30 to 0.70 and g-variation between .37 to 1. In this case also the robust passification condition was found to be conservative. The plant uncertainty ($\Delta(P)$) was modeled as additive perturbation by looking at the largest difference between the magnitude plots of the nominal model (Model 2) and perturbed models (Models 1, 3, and 4). As seen in Figure 12, the passification was found to be robust for all four flight conditions (as indicated by phase remaining between $\pm 90^\circ$). However, the robustness condition (not shown) of Theorem 1 was violated, i.e., the $\overline{\sigma}(\Delta P)$ could not satisfy the inequality of Eq.(1) in certain frequency range. Having robustly passified the plant, a short-period approximation of the plant was used as the design model for the plant. An LQG-optimal, fifth-order WSPR controller [Loz.90a] was then designed for the design flight condition to obtain satisfactory response. For detailed description on the controller design please refer to [Kel.97].

4 Concluding Remarks

Robust control of non-passive systems via passification was considered. A number of sufficient conditions were obtained for robust passification of non-passive linear, time-invariant systems using series, feedback, and feedforward passification, in the presence of additive, multiplicative, and feedback uncertainties. These conditions can be used to check robustness of passification or for iterative controller design. The results were demonstrated by application to rotational control of a planar elastic beam and longitudinal control system design for a fighter aircraft model. For the elastic beam example, the robust passification condition was satisfied for additive perturbation. For unmodeled dynamics and parametric uncertainty in the design model (modeled together as additive perturbation), the same condition was found to be too conservative. Robust passification conditions in the presence of simultaneous perturbation models are not available at present and future work should address this problem. In the case of the fighter aircraft example, the phase plots showed that robust passification was achieved for all four flight conditions that were considered; however, the robustness condition was not satisfied. In summary, the conditions presented offer a useful tool in the design of robust controllers for non-passive systems using passification techniques proposed in [Kel.97]. However, these conditions are conservative and further research is necessary to obtain weaker conditions.

5 References

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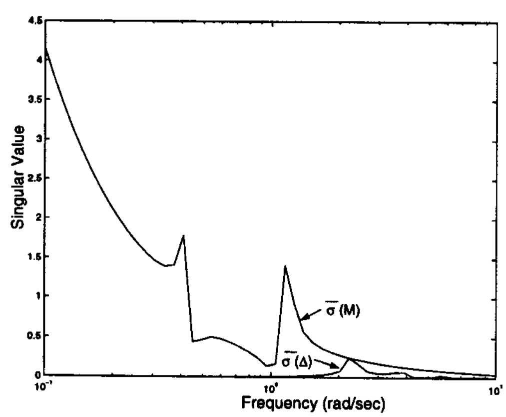

Figure 10: Robustness test for Feedback passification with additive uncertainty

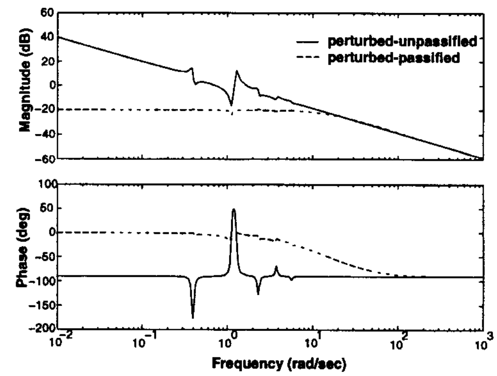

Figure 11: Bode plots of passified and unpassified plants

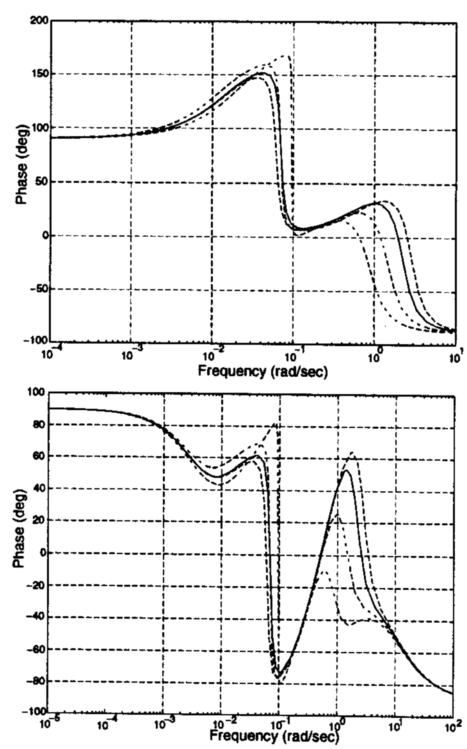

Figure 12: Phase plots of unpassified (top) and passified (bottom) systems